

**Instructions to be read out loud to participants before beginning of examination**

1. Please do not open examination booklet until directed to do so.
2. Please turn all cell phones, blackberries, etc. for the duration of the examination.
3. You may not use any materials (calculators, study notes, cell phones, etc.) during the examination.
4. If you have a question, raise your hand and a proctor will come to assist you. You may not speak to the proctor until you are out of hearing range of other contest participants.
5. If you need to use the restroom facilities, request permission from the proctor and go quietly but do not take any material with you.
6. The test consists of two parts:
  - part 1-- 6 multiple choice short answer problems,  
and
  - part 2-- 4 long answer problems .
7. Please mark the answers to the 6 short answer problems using the scantron cards and pencils provided to you. Make sure your full name is written on your scantron card and test booklet.
8. The top 6 scorers on the short answer portion of the examination shall be declared "contest finalists" and have their long answer portion of their examination graded. Please remember that the questions on this examination are fairly difficult and it is unlikely that many students will get them all correct. You probably do not need to score 100% (or even close to 100%) to win an examination prize.
9. The long answer problems must provide detailed written solutions to the problems. Partial credit will be provided when justified.
10. For the purpose of awarding the prizes to the top 3 finalists, each of the 6 short answer problems will count for 6 points and each of the 4 long answer problem will count for 16 points. Thus a participant with a perfect paper will score a 100.
11. Any participant who violates any of the test rules or is disruptive to others will be disqualified, and their paper will not be graded.
12. Open your booklets and begin your examination. The time is now \_\_\_\_\_. You have two hours to complete this examination. The proctor will announce to you when 1 hour remains, when 30 minutes remain and when 5 minutes remain. Please remember that any writing after the examination time is expired will disqualify your paper.

**Instructions to be read out loud to participants at end of examination**

1. Time is expired. **Please stop writing now!**
2. Please check that your name is on the scantron card as well as on your booklet. If it is not, you may write your name now.
3. The proctor will collect your question sheet, booklet and scantron now.
4. Please wait until all papers are collected before leaving
5. Thank you for your participation in this contest.

# Examination Booklet

Participant name \_\_\_\_\_

Participant School \_\_\_\_\_

## *Multiple Choice*

*For the multiple choice questions you need not show any work. No partial credit will be given. Clearly mark your answer (a, b, c, d, or e) to each question on the scantron card using pencil provided. If you need to erase, erase completely. Any ambiguous markings will be scored as incorrect. No calculators may be used.*

**Problem 1)**

1) If  $a=105$  is the first number in a primitive Pythagorean triple  $(a,b,c)$ , then how many different primitive Pythagorean triples can be arranged?

- a) 1
- b) 2
- c) 3
- d) 4
- e) 6

**Problem 2)**

What is the third number in a primitive Pythagorean triple  $(a,b,c)$ , which is defined by coordinates of the intersection point  $(x,y)$  of the unit circle with the center in the origin and the line with the slope  $m=0.4$  passing through  $(-1, 0)$  and  $(x,y)$ ?

- a) 25
- b) 27
- c) 29
- d) 31
- e) 33

**Problem 3)**

What is the remainder when we divide  $11^{11}-4^{10}$  by 5?

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

**Problem 4)**

What is the minimal positive solution of the equation  $3^{121+x} \equiv 2 \pmod{7}$ ?

a) 0

b) 1

c) 2

d) 3

e) 4

**Problem 5)**

A number  $L$  is called a common multiple of  $m$  and  $n$  if both  $m$  and  $n$  divide  $L$ . The smallest such  $l$  is called the least common multiple of  $m$  and  $n$  and is denoted by  $\text{LCM}(m,n)$ . How many pairs  $(m,n)$  have  $\text{gcd}(m,n)=18$  and  $\text{LCM}(m,n)=720$  ? (pairs  $(m,n)$  and  $(n,m)$  are considered as the same ones)

- a) 1
- b) 2
- c) 3
- d) 4
- e) 5

**Problem 6)**

What is the last digit of the number  $2^{999}$ ?

a) 0

b) 2

c) 4

d) 6

e) 8

## *Long Answers*

*For the long answers you must show all work. Partial credit will be given as appropriate. No credit will be given **if** work is not shown. No calculators may be used. (Write your answers on the question sheets)*

### **Problem 7)**

Using Euclidean Algorithm, find  $\gcd(8277, 9951)$ .

**Problem 8)**

Prove that  $a^3 \equiv a \pmod{6}$ .

**Problem 9)**

Solve the equation  $689x \equiv 106 \pmod{1643}$

**Problem 10)**

Describe all integer solutions of the equation  $407x+259y=629$ .