Multiple Choice

For the multiple choice questions you need not show any work. No partial credit will be given. Clearly mark your answer (a, b, c, d, or e) to each question on the scantron card using pencil provided. If you need to erase, erase completely. Any ambiguous markings will be scored as incorrect. <u>No calculators may be used.</u>

Problem 1)

Forty books are placed on a shelf, among them the three volumes of "The Seventh Column" by Alterman. Find the probability that the three volumes are arranged on the shelf according to the correct order (not necessarily adjacent to each other).

(a) $\frac{38 \cdot 39 \cdot 40}{6 \cdot 40!}$

(b) 1/40

(c) $\frac{1}{40 \cdot 6}$

● 1/6

(e) 6/40

Problem 2)

If a test consists of 15 true-false questions, in how many different ways can a student mark his test paper if he is allowed to leave blank **at most** 3 questions?

(a) $2^{12}*(576)$
(b) $2^{12}*(733)$
(c) $2^{15}*(376)$
(d) $2^{15}*(576)$
(e) $2^{15}*(733)$

Problem 3)

Suppose that a jar contains *2N* cards, two of them marked 1, two marked 2, two marked 3, and so on. Draw out *m* cards at random. What is the expected number of pairs that still remain in the jar?

(a) $\frac{(2N-m)^2}{(2N-1)}$

(b) $\frac{(2N-m)(2N-m-1)}{2N(2N-1)}$

(c) $\frac{(2N-m+1)(2N-m-1)}{(4N)}$

(d) $\frac{N(2N-m-1)}{2(2N-1)}$

● $\frac{(2N-m)(2N-m-1)}{2(2N-1)}$

Problem 4)

Ten hunters are waiting for ducks to fly by. When a flock of ducks flies overhead, the hunter's fire at the same time, but each chooses his target at random independently of the others. If each hunter independently hits his target with probability *p*, compute the expected number of ducks that escape unhurt when a flock of size 10 flies overhead.

● $10(1-\frac{p}{10})^{10}$

(b) $10(\frac{p}{10})^{10}$

(c) $10(1-p)^{10}$

(d) $(\frac{p}{10})^{10}$

(e) $(1-\frac{p}{10})^{10}$

Problem 5)

You write individual letters to 12 of your friends and address 12 envelopes, one for each letter. For fun, you shuffle the envelopes, close your eyes, and then randomly stuff one letter into each of the envelopes. What is the probability that precisely 8 of the envelopes contain the correct letters?

(a) $\frac{3960}{8!}$

(b) $\frac{4455}{8!}$

(c) $\frac{3960}{12!}$

(d) $\frac{4455}{12!}$

(e) $\frac{5940}{12!}$

Problem 6) Find the sum of:

$\frac{1}{3\cdot4\cdot5}+\frac{2}{4\cdot5\cdot6}+\frac{3}{5\cdot6\cdot7}+\ ...+\frac{k}{(k+2)\cdot(k+3)\cdot(k+4)}$ when **n = 100.**

(a) 0.137
(b) 0.147
● 0.157
(d) 0.177
(e) 0.187

Long Answers

You must show all work. Partial credit will be given as appropriate. No credit will be given if work is not shown. No calculators may be used.

Problem 7)

Many door locks use doors with 5 buttons numbered from 1 to 5. Legal combinations allow the buttons to be pushed in specific order either singly or in pairs without pushing any button more than once. Thus [(12), (34)] = [(21), (34)]; [(1), (3)]; and [(2), (13), (4)] are legal combinations while [(1), (14); (134)]; and [(13), (14)] are not. How many legal combinations are there? Show all work below or on the other side of this page.

Problem 8)

What is the greatest integer the expression $n^9 - n$ divisible by, for all integers $n > 0$.

Show all work below or on the other side of this page.

Problem 9)

What is the probability that at least two students in a group of k students have the same birthday (month and day)? Show all work below or on other side of this page.

Problem 10)

Prove that $n! \geq 2^n$ for all natural numbers n, where $n \geq 4$. Show all work below or on the other side of this page.